

Please check the examination details below before entering your candidate information

Candidate surname

Other names

**Pearson Edexcel  
Level 3 GCE**

Centre Number

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Candidate Number

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**Tuesday 25 June 2019**

Morning (Time: 1 hour 30 minutes)

Paper Reference **9FM0/4A**

**Further Mathematics**

**Advanced**

**Paper 4A: Further Pure Mathematics 2**

**You must have:**

Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B)
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

### Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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Answer ALL questions. Write your answers in the spaces provided.

1. A complex number  $z = x + iy$  is represented by the point  $P$  in an Argand diagram.

Given that

$$|z - 3| = 4|z + 1|$$

- (a) show that the locus of  $P$  has equation

$$15x^2 + 15y^2 + 38x + 7 = 0$$

(2)

- (b) Hence find the maximum value of  $|z|$

(3)

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2. The matrix  $\mathbf{A}$  is given by

$$\mathbf{A} = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$$

- (a) Show that 2 is a repeated eigenvalue of  $\mathbf{A}$  and find the other eigenvalue. (5)
- (b) Hence find three non-parallel eigenvectors of  $\mathbf{A}$ . (4)
- (c) Find a matrix  $\mathbf{P}$  such that  $\mathbf{P}^{-1}\mathbf{A}\mathbf{P}$  is a diagonal matrix. (2)

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3. The number of visits to a website, in any particular month, is modelled as the number of visits received in the previous month plus  $k$  times the number of visits received in the month before that, where  $k$  is a positive constant.

Given that  $V_n$  is the number of visits to the website in month  $n$ ,

- (a) write down a general recurrence relation for  $V_{n+2}$  in terms of  $V_{n+1}$ ,  $V_n$  and  $k$ . (1)

For a particular website you are given that

- $k = 0.24$
- In month 1, there were 65 visits to the website.
- In month 2, there were 71 visits to the website.

- (b) Show that

$$V_n = 50(1.2)^n - 25(-0.2)^n \quad (5)$$

This model predicts that the number of visits to this website will exceed one million for the first time in month  $N$ .

- (c) Find the value of  $N$ . (2)

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5.

$$I_n = \int \operatorname{cosec}^n x \, dx \quad n \in \mathbb{Z}$$

(a) Prove that, for  $n \geq 2$ 

$$I_n = \frac{n-2}{n-1} I_{n-2} - \frac{\operatorname{cosec}^{n-2} x \cot x}{n-1} \quad (4)$$

(b) Hence show that

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \operatorname{cosec}^6 x \, dx = \frac{56}{135} \sqrt{3} \quad (4)$$

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6. (i) A binary operation  $*$  is defined on positive real numbers by

$$a * b = a + b + ab$$

Prove that the operation  $*$  is associative.

(4)

- (ii) The set  $G = \{1, 2, 3, 4, 5, 6\}$  forms a group under the operation of multiplication modulo 7

(a) Show that  $G$  is cyclic.

(2)

The set  $H = \{1, 5, 7, 11, 13, 17\}$  forms a group under the operation of multiplication modulo 18

(b) List all the subgroups of  $H$ .

(3)

(c) Describe an isomorphism between  $G$  and  $H$ .

(3)

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8.

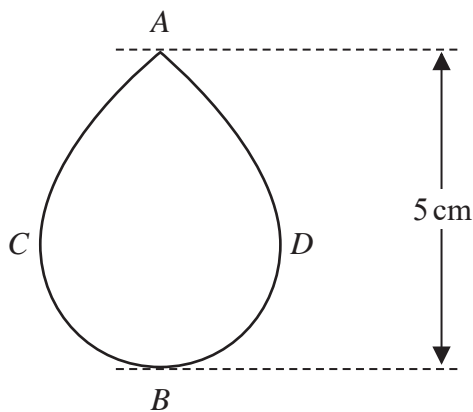


Figure 1

Figure 1 shows the vertical cross section of a child's spinning top. The point  $A$  is vertically above the point  $B$  and the height of the spinning top is 5 cm.

The line  $CD$  is perpendicular to  $AB$  such that  $CD$  is the maximum width of the spinning top.

The spinning top is modelled as the solid of revolution created when part of the curve with polar equation

$$r^2 = 25 \cos 2\theta$$

is rotated through  $2\pi$  radians about the initial line.

(a) Show that, according to the model, the surface area of the spinning top is

$$k\pi(2 - \sqrt{2}) \text{ cm}^2$$

where  $k$  is a constant to be determined.

(7)

(b) Show that, according to the model, the length  $CD$  is  $\frac{5\sqrt{2}}{2}$  cm.

(6)

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